Lecture 18

- Frequency/phase noise and linewidth

The Schawlow-Townes linewidth

- In addition to the intensity noise, the laser also produces frequency noise, which affects the laser spectral linewidth. DFB or DBR single mode lasers usually have linewidths of several megahertz.
- The phase noise induced linewidth arise from two basic sources: (1) spontaneous emission and (2) carrier density fluctuations.
- The first is inherent in all lasers, resulting from the random addition of spontaneously emitted photons to the quasi-coherent resonant cavity modes.
- The second is of significance only in diode lasers, and it results the phase-amplitude (gain) coupling effect described by the linewidth enhancement factor.

The Schawlow-Townes linewidth

- In a cavity without gain medium but with photons, the photon change

$$\frac{dN_P}{dt} = -\frac{N_P}{\tau_p} \Rightarrow N_P(t) = N_{P0} e^{-t/\tau_p}$$

- The corresponding electric field is (u(t) is a step function)

$$E(t) = A_0 e^{j\omega_0 t} e^{-t/2\tau_p} u(t)$$

- The frequency domain response of the laser cavity (cold cavity response) is given by the Fourier transform, which is of a Lorentzian shape,

$$\left|E(\omega)\right|^2 = \frac{\left|E(\omega_0)\right|^2}{1 + \left(\omega - \omega_0\right)^2 (2\tau_p)^2}$$

上海科技大学
ShanghaiTech University

The Schawlow-Townes linewidth

- Thus, the FWHM spectral linewidth of the cold cavity corresponds to the filter bandwidth of the FP resonator mode with no active medium,

$$\Delta\omega = \frac{1}{\tau_P}$$

- Now, we add back the stimulated term that is responsible for gain in the cavity, the effective cavity lifetime is

$$\frac{1}{\tau_P'} = \frac{1}{\tau_P} - \Gamma v_g g \Rightarrow$$

$$\Delta\omega = \frac{1}{\tau_P'}$$

上海科技大学 ShanghaiTech University

The Schawlow-Townes linewidth

- The spontaneous emission induced spectral linewidth is

$$\Delta v_{spon} = \frac{1}{2\pi\tau_P{}'} = \frac{\Gamma R_{sp}^{'}}{2\pi N_p}$$

(Schawlow-Townes linewidth, below threshold)

- The above-threshold spectral linewidth is modified as

$$\Delta v_{ST} = \frac{\Gamma R_{sp}^{'}}{4\pi N_p}$$

(modified Schawlow-Townes linewidth, above threshold)

- Note that the ST linewidth only considers the spontaneous emission noise and does not include carrier noise.

Frequency noise

- The phase variation of the laser is given by

$$\frac{d\phi}{dt} = 2\pi\Delta v(t) = \frac{\alpha}{2}\Gamma v_g a\Delta N(t) + F_\phi(t)$$

- In the frequency domain,

$$v_1(\omega) = \frac{\alpha}{4\pi}\Gamma v_g a N_1(\omega) + \frac{1}{2\pi}F_\phi(\omega)$$

- The correlation strengths for the phase noise are

$$\left\langle F_\phi F_\phi \right\rangle = \frac{\Gamma R_{sp}^{'}}{2N_p}, \ \left\langle F_\phi F_P \right\rangle = \left\langle F_\phi F_N \right\rangle = 0$$

- The phase noise is uncorrelated with photon and carrier density noises

Frequency noise (another way)

- The phase rate equation,

$$\frac{d\varphi}{dt} = \frac{\alpha}{2}\left(\Gamma v_g g - \frac{1}{\tau_p}\right) + F_\phi(t)$$

- The small-signal analysis of the phase rate equation,

$$\frac{d}{dt}(d\varphi) = \frac{\alpha}{2}\Gamma v_g dg + F_\phi(t) \Rightarrow$$

$$\frac{d}{dt}(d\varphi) = \frac{\alpha}{2}\Gamma v_g \left(adN - a_p dN_p\right) + F_\phi(\omega)$$

$$j\omega\varphi_1 = \frac{\alpha}{2}\Gamma v_g \left(aN_1 - a_p N_{p1}\right) + F_\phi(\omega)$$

上海科技大学 ShanghaiTech University

Frequency noise (another way)

- The differential rate equations,

$$\begin{bmatrix} \gamma_{NN} + j\omega & \gamma_{NP} & 0 \\ -\gamma_{PN} & \gamma_{PP} + j\omega & 0 \\ -\frac{\alpha}{2}\Gamma v_g a N_1 & \frac{\alpha}{2}\Gamma v_g a_p N_{p1} & j\omega \end{bmatrix} \begin{bmatrix} N_1 \\ N_{p1} \\ \varphi_1 \end{bmatrix} = \begin{bmatrix} F_N(\omega) \\ F_P(\omega) \\ F_\varphi(\omega) \end{bmatrix}$$

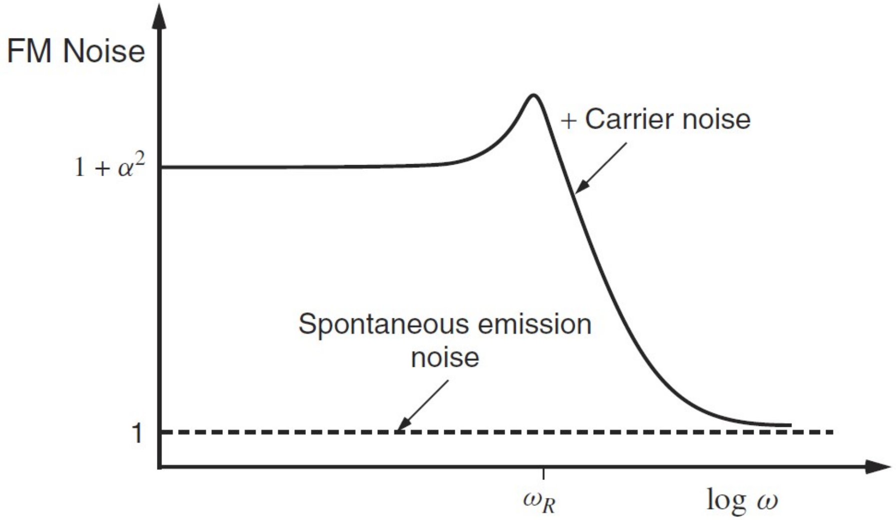
- The frequency noise is given by

$$\boxed{FN(\omega) = |v_1(\omega)|^2 = \left| \frac{j\omega}{2\pi} \varphi_1(\omega) \right|^2}$$

The frequency noise

- The frequency noise spectral density is given by (double-sided)

$$FN(\omega) = \frac{1}{2\pi}(\Delta v)_{ST}\left(1+\alpha^2\left|H(\omega)\right|^2\right)$$



- At low frequency, the total optical linewidth is given by

$$\begin{aligned}\Delta v_{OL} &= 2\pi FN(\omega\to 0)\\ &= (\Delta v)_{ST}\left(1+\alpha^2\right)\end{aligned}$$

- At high frequency, the Shawlow-Townes linewidth is given by

$$(\Delta v)_{ST} = 2\pi FN(\omega\to\infty)$$

上海科技大学 ShanghaiTech University

Optical linewidth

- The autocorrelation of the light can be described by the coherence time,

$$\left\langle E(t)E(t-\tau)^{*}\right\rangle \propto e^{j\omega\tau}e^{-\tau/\tau_{coh}}$$

- The first factor give the expected interference fringes created by the coherent mixing. However, because the laser is not emitting a pure single frequency, with increasing time delay tau, the phases of the two fields become less and less correlated and the interference fringes gradually disappear to the point where the two fields add incoherently. The envelope of the fringe pattern given by the second factor characterizes this coherent decay, which is described by the coherence time tau_coh.

Coherent time

- The coherent time can be measured by mixing the electric field E(t) with a time-delayed version of itself E(t-tau),

$$e^{-\tau/\tau_{coh}} = \frac{P_1 + P_2}{2\sqrt{P_1 P_2}} \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}}$$